Some properties and applications of the moiré effect obtained with dots distributions

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Abstract

Moiré patterns for dots or particles with two-dimensional random distribution are obtained through the superposition of an original distribution and another one is obtained from the same distribution applying self-affine transformations. This way, different structures can be obtained such as circles, ellipses, spirals, etc.. In this paper, the first result involves some properties of the cross-correlation function of the structures. The second result refers to the possibility of obtaining moiré patterns from the evolution of a biological textures along time and the characterization with the cross-correlation function.

1. Introduction

The formation of moiré patterns from the superposition of random dots distribution is a topic already studied by many authors [1-6]. When an original image of granular structure (periodic or random) is superimposed with another image that is obtained by means of a self-affine transformation [6], from the original one, connections are obtained between the granular elements of both images. Such structure is of the same nature that the moiré effect obtained in the superposition of grids (periodic or random) [7-11].

Other applications related with these facts are the re-crystallization kinetics process, and some authors are interested in the modelling of periodic and random lattices [12-14]. Also, some spiral lattices that are illustrated in composite flowers have concentric crystalline grains that can be studied with this moiré effect.

Here, we are interested in the pattern recognition applications of the obtained moiré structures and we study some properties for these distributions using the cross-correlation function. A fully desirable pattern-recognition device should be able to recognize any given design, regardless of (a) its position on the plane, (b) its orientation in the plane, or (c) its size or scale of magnification. In this work, we want to establish the basis for a new pattern-recognition device (soft+hard), which can be developed from the results obtained in this paper.

Then, we obtain some laws for the cross-correlation function for two or more superimposed random images of dots. The correlation between the original image, with its magnified version, with and without rotation between them is measured. First image is the original and the second image is the transformed from the original one (with a self-affine transformation). Then, circles, radial lines or spirals are obtained from the superposition between them. As a second study of these facts we show the possibility for studying the evolution of biological surfaces (for example an orange fruit, in this case) through the cross-correlation of the moiré pattern, which is formed from the superposition of the texture of the surface for two different days. Both texture images have irregularities that are registered as gray levels, this constitutes an application for the superposition of dots distributions.

2. Theory of the moiré effect

The well known moiré effect [7-11] is obtained from the superposition of two grids. The top image in Fig. 1 shows the pattern of clear and dark fringes which are obtained in the superposition of periodic grids, while the bottom image has a single peak of intensity in the case of random grids. Many applications have been developed, considering the moiré obtained with periodic or random grids [15-18]. Another type of application has to do with the digital halftoning [19-21]. We should also highlight that the moiré pattern is a measurement of the correlation between two grids or dots distributions [9].

Mathematically, the result for the case of grids defined as:

$$T_k(x,y) \;=\; \sum_{n=-\infty}^{+\infty} C_n \exp\left[\frac{2\pi\, i}{d_k}\, n\, \Phi_k(x,y)\right] \qquad (1)$$
$$k = 1, 2 \quad \text{and} \quad \Phi_k(x,y) = x\cos\theta \pm y\sin\theta$$

where d_1 and d_2 are the periods of each transmittance and θ is the relative angle, can be finally expressed through the product of the functions representing the grids (T_1 and T_2), this is:

$$\mathrm{M}_{12}(x,y) = \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} C_n\, C_m \exp\left[2\pi\, i\, \Delta\right]$$
$$\Delta = \frac{n}{d_1}\Phi_1(x,y) + \frac{m}{d_2}\Phi_2(x,y) \qquad (2)$$

This result can be extended for random grids but, in this case only one peak of cross-correlation is obtained as shown the bottom image of Fig. 1 (central line), as we previously pointed out.

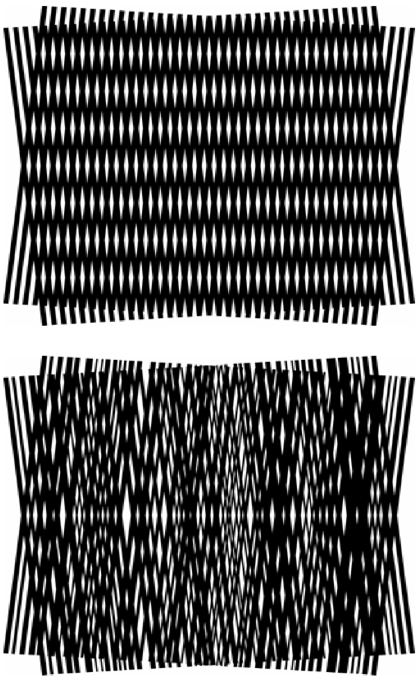


Figure 1. Moiré effect for two grids with an angular displacement between them: (a) periodic, (b) random.

Because the moiré phenomenon is also presented for random grids, the results can be extended for the case of dots or particles. The results are shown in the Fig. 2, in the superposition between a texture and an identical copy of itself with an angular displacement (top image). In this case a circular structure is obtained. Also, radial lines are obtained when we superimpose the initial distribution and a copy with a linear magnification (or contraction), as shown in the central image. Finally, from the superposition of the initial distribution and a magnified copy, with an angular rotation between both, we obtain spirals (bottom image).

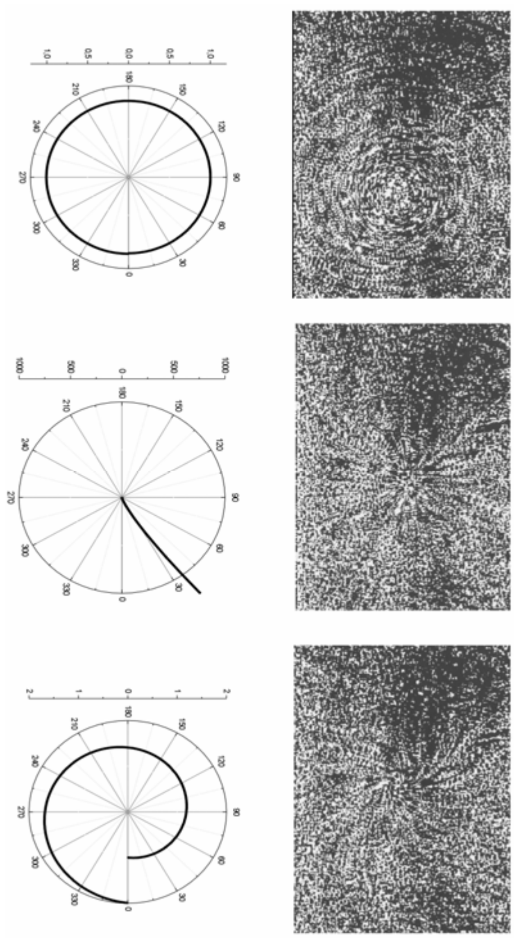


Figure 2. Moiré patterns (from top to bottom): circular, radial and spiral.

2.1. Cross-correlation function

The cross-correlation function [22] permits us to relate the final structure obtained with the class of transformation applied. The cross-correlation between a function (f) and a transformed version of itself (f^T) is expressed as:

$$f(x,y) * f^T(x,y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x',y')\, f^T(x'-x, y'-y)\, dx'\, dy' \qquad (3)$$

in rectangular coordinates (x,y). An application, using the correlation and fractal dimensions, for the classification of galaxies (with similar structures as shown in Fig. 1), has already been developed [23]. Our work is related with these results in the sense that circles, radial lines and spirals obtained from the superposition of two random distributions (with a self-affine transform between them) can be characterized through the correlation and fractal dimensions. As previous work in this direction can be seen Ref. [4].

In the case of the moiré patterns from Fig. 1 the calculus of the cross-correlation function is made along the transversal coordinate to the fringe (vertical direction), and can be expressed as [8]:

$$P_c(x) = \lim_{T\to\infty} \frac{1}{T} \int_{-T}^{+T} M_{12}(x,y)\, dy$$
$$\Rightarrow P_c(i) = \frac{1}{N} \sum_{j=1}^{N} I(i,j) \qquad (4)$$

being (i,j) pairs of integer numbers indicating the position of pixels (in Cartesian sense) in the image and N is the number of pixels in the direction of fringes.

The result of Eq. (4) can extrapolated for other coordinate system, for example polar coordinates, and then:

$$C_\theta(r) = \lim_{T\to\infty} \frac{1}{T} \int_{-T}^{+T} S(r,\theta)\, d\theta$$
$$\Rightarrow C_\theta(i_r) = \frac{1}{K} \sum_{j=1}^{K} S(i_r, j_\theta) \quad \textit{for circles}, \qquad (5)$$

$$C_r(\theta) = \lim_{T\to\infty} \frac{1}{T} \int_{-T}^{+T} S(r,\theta)\, dr$$
$$\Rightarrow C_r(j_\theta) = \frac{1}{M} \sum_{j=1}^{M} S(i_r, j_\theta) \quad \textit{for radial lines}.$$

being now C_θ and C_r the cross-correlations along the radial and the angular direction respectively, and K, M are the number of pixels along the transversal direction to the moiré profiles. The functions S indicates the image obtained from the superposition of two or more distributions of particles or dots, and the pair of integer (i_r,j_θ) indicates the position of each pixel (in polar sense).

3. Laws for the cross-correlation function

The degree of correlation between two images with a (self-affine) transformation between them depends on some important facts, and we can obtain interesting conclusions when some features are taken into account [3]:

1- Change in the statistics of the distribution of systems (for the original and transformed images).
2- The elements must be different in size to obtain a higher degree of correlation and this correlation depends largely on the number of these elements included in the image.

Also, we can find the following properties:

1- When three or more images are superimposed with a relative scale factor between them, we can observe that the correlation is gradually variable and then, we can take a bigger interval in the scaling factors (see Fig. 3).
2- When we change the gray level of the transformed distribution, the cross-correlation depends also on this variable. This result is related with the binary characteristic of both distributions, which permits the formation of moiré patterns.

The property 1 relates directly with the density of particles obtained by different scaling, and the structures are more clears, just as can be seen in Fig. 3. The property 2 is very interesting, and it is illustrated in Fig. 4, where the radial lines and circles structures can be clearly observed, for gray levels and black distribution superimposed with a transformed copy with equal gray level. Contrarily, when both distributions have different gray levels it is more difficult to visualize the moiré structures (see central images). Such result can be seen graphically in Fig. 5, since for the case of two distributions of particles or dots with black color (binary images), the cross-correlation function is visually established between the positions indicated in the Fig. 5a, while when there are different gray levels, the visual cross-correlation is established in a longer distance as show Fig. 5b. We should clarify that this happens fundamentally when the whole dots distribution has a certain gray level and the whole (self-affine) transformed distribution has a different gray level. This distance for the visual cross-correlation has an effect on the calculated cross-correlation: this result has a minor value than the case when both dots or particles distributions have the same gray levels.

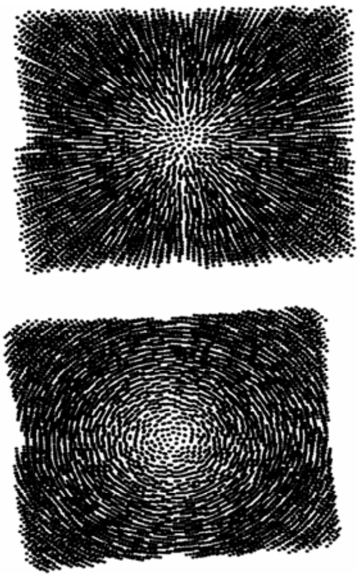


Figure 3. Superposition of five dot distributions.

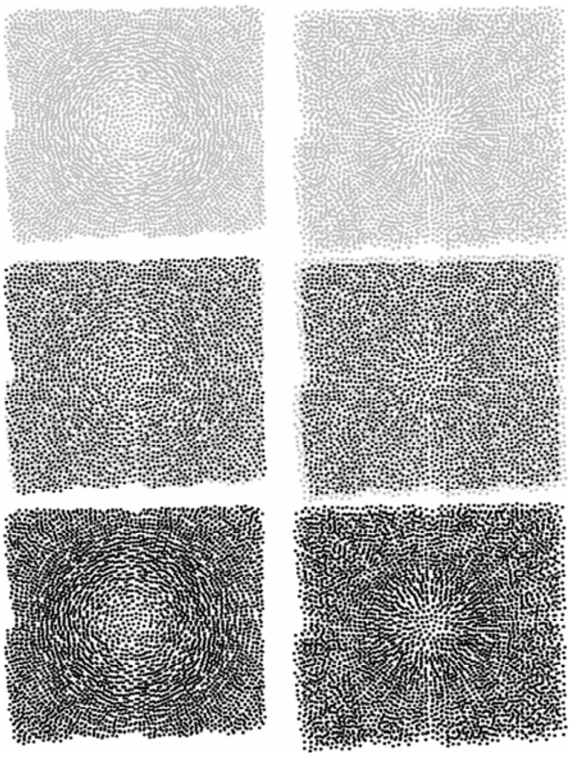


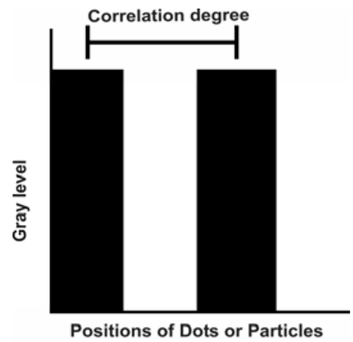
Figure 4. Formation of radial lines and circles for the superposition of distributions (original and transformed) with equal gray level (above), gray level and black (middle) and black (bottom).

In the evolution of the moiré pattern formed in the superposition of two self-affine dots distributions it is observed that the structures obtained (circles, radial lines) depends on the angle of rotation between both distributions. Then, an appropriate form of characterizing the structures is through the evolution of the gyration ratio (R_g), which is given by [24]:

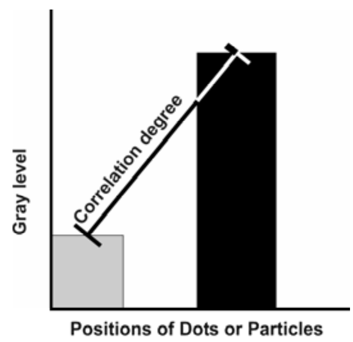
$$R_g^2 = \frac{1}{N}\sum_{k=1}^{N}\left(r_k - <r>\right)^2 \quad , \quad <r> = \frac{1}{N}\sum_{k=1}^{N} r_k \quad (6)$$

where r_k is the position of each particle or dot and N the number of dots into de structure, $<r>$ is the mean ratio.

Table 1 shows the angular cross-correlation obtained for the case of circles, along the gyration ratio (central column), as well as the angular cross-correlation obtained with the first equality of Eqs. (5), using the whole image. We should mention that this correlation also depends strongly on the density of points, just as already pointed out for Fig. 3.



(a)



(b)

Figure 5. Illustration of the visual correlation degree.

4. Applications for biological surfaces

It is very simple to visualize the effects before mentioned in different biological distributions for studying their evolution along time. There are many possible applications to develop in this sense, involving biological systems. The images that we show next refer to the process of degradation of an orange fruit.

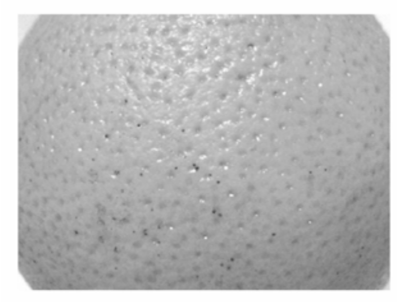
Due to the lost of water in the peel, the orange suffers small modifications along time, which can be visualized day to day and so, follow the evolution of the texture. This way, images of the orange texture can be superimposed to obtain moiré structures and, this way, to be able to measure the cross-correlation between the texture for a certain day and the corresponding texture for an initial day. Then, structures with radial lines, circles or spirals can be obtained, just as it is shown in Fig. 6.

Then, moiré effect in bio-textures can be used as a tool for studying their evolution and its properties. But we should highlight that the phenomenon is sensitive only for small changes, which could describe slow biological processes and superimposing images with small temporal difference between them.

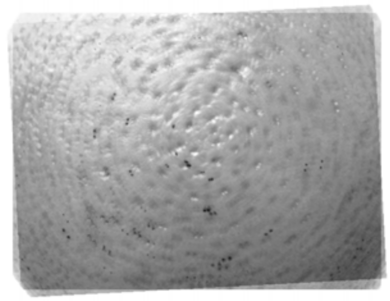
Table 2 shows the results obtained for the calculated cross-correlation on the gyration ratio, when superimposing the texture of an orange for two different days. Two distributions with a rotation between them were considered in such case and then, spirals were always obtained, except for the first day when were obtained the circles. Maximum and minimum radios were estimated and the gyration ratio calculated in this region. The cross-correlation function was calculated also for inside part this region.

Gray levels of dots	Cross-correlation at R_g	Correlation for the image
	0.9021	0.1002
	0.7790	0.0823
	0.2675	0.0350
	0.0411	0.0129

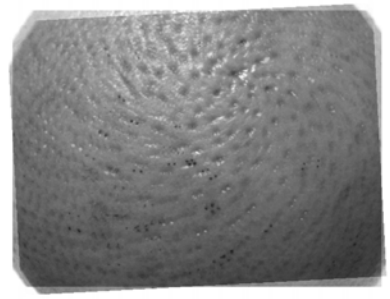
Table 1. Angular cross-correlation between dots distributions with different gray scales and black distribution of dots.



(a)



(b)



(c)

Figure 6. Evolution of the texture of an orange fruit: (a) initial texture, superposition of the initial texture with: (b) a rotated copy of itself, (c) a rotated image of itself obtained several days later.

DAYS	CROSS-CORRELATION	GIRATYON RATIO (in pixels)
1 – 1	0.28758	282
1 – 3	0.24837	181
1 – 8	0.154967	154
1 – 13	0.073072	99

Table 2. Cross-correlation of the surface orange for different days.

5. Conclusions

In this work we show two interesting results regarding the moiré pattern formation from the superposition of two images with random distributions of dots or particles. The first result is related with the properties of the cross-correlation function, involving gray levels. We should clarify that such distributions are the original and another one obtained from the original, but applying self-affine transformations on itself. The second result is an application for the evolution of biological textures. In this case, the evolution is a slow process and we superimpose two images obtained at two different times. The results shows that the cross-correlation for the superposition of two dots or particles distributions is very useful to characterize the structure obtained (circles, radial lines, etc.), in relation to the observed visual correlation.

For future studies other forms or structures will be taken into account, which can appear, such as radial lines from a segment, hyperbolas, ellipsis, and three-dimensional structures studied along cutting planes in different directions. All these cases will allow us to extend this class of studies to take into account stress and pressures which can be acting in the evolution of the biological system.

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